*AAT (IC/HW)-Day 1

Name Key

Chapter 10: 10-1 Infinite Sequences and Summation Notation

Date: __________ Period: __________

Find the first four terms and the eighth term of the sequence.

1. $\{12-3n\}$

$a_1 = 12-3(1) = 9$

$a_2 = 12-3(2) = 6$

$a_3 = 12-3(3) = 3$

$a_4 = 12-3(4) = 0$

$a_8 = 12-3(8) = -12$

2. $\{\frac{3n-2}{n^2+1}\}$

$a_1 = \frac{1}{2}$

$a_2 = \frac{4}{5}$

$a_3 = \frac{7}{10}$

$a_4 = \frac{10}{17}$

$a_8 = \frac{22}{65}$

3. $\{9\}$

$a_1 = 9$

$a_2 = 9$

$a_3 = 9$

$a_4 = 9$

$a_8 = 9$

4. $\{(-1)^{n-1}\frac{n+7}{2n}\}$

$a_1 = 4$

$a_2 = -\frac{9}{4}$

$a_3 = \frac{5}{3}$

$a_4 = -\frac{11}{8}$

$a_8 = \frac{-15}{16}$

5. $\{1+(-1)^{n+1}\}$

$a_1 = 2$

$a_2 = 0$

$a_3 = 2$

$a_4 = 0$

$a_8 = 0$

6. $\{\frac{2^n}{n^2+2}\}$

$a_1 = \frac{2}{3}$

$a_2 = \frac{2}{3}$

$a_3 = \frac{8}{11}$

$a_4 = \frac{8}{9}$

$a_8 = \frac{128}{33}$

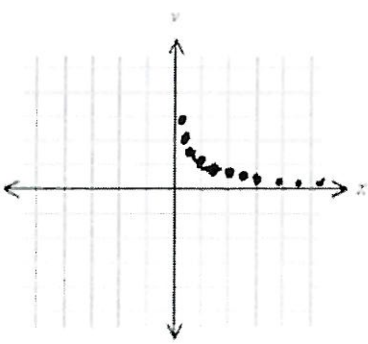
7. a_n is the number of decimal places in $(0.1)^n$.

$\{.1, .01, .001, .0001; .00000001\}$

1, 2, 3, 4, 8 $\rightarrow$ # of decimal places

Graph the sequence.

8. $\{\frac{1}{\sqrt{n}}\}$

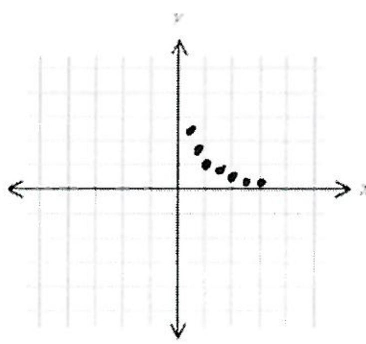


$\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}, \frac{1}{\sqrt{5}}, \ldots$

$\approx 1, .71, .58, .5, .45, \ldots$

$(1,1), (2,.71)\ (3,.58) \ldots$

9. $\{\frac{1}{n}\}$

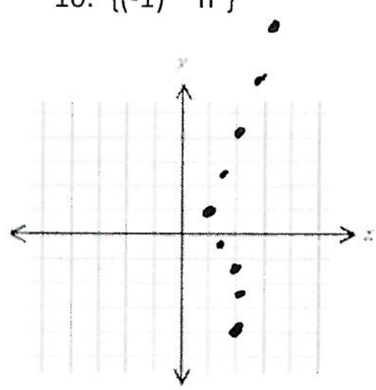


$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

$= 1, .5, .\overline{3}, .25, .2, \ldots$

$(1,1), (2,.5), (3,.\overline{3}) \ldots$

10. $\{(-1)^{n+1}n^2\}$



$1 \cdot 1^2, -1 \cdot 2^2, 1 \cdot 3^2, -1 \cdot 4^2, \ldots$

$1, -4, 9, -16, \ldots$

$(1,1), (2,-4), (3,9), (4,-16) \ldots$

Find the first five terms of the recursively defined infinite sequence.

11. $a_1 = 2, \quad a_{k+1} = 3a_k - 5$

$a_2 = 3(2) - 5 = 1$

$a_3 = 3(1) - 5 = -2$

$a_4 = 3(-2) - 5 = -11$

$a_5 = 3(-11) - 5 = -38$

12. $a_1 = -3, \quad a_{k+1} = a_k^2$

$a_2 = (-3)^2 = 3^2$

$a_3 = (3^2)^2 = 3^4$

$a_4 = (3^4)^2 = 3^8$

$a_5 = (3^8)^2 = 3^{16}$

13. $a_1 = 5, \quad a_{k+1} = ka_k$

$a_2 = 1(5) = 5$

$a_3 = 2(5) = 10$

$a_4 = 3(10) = 30$

$a_5 = 4(30) = 120$

14. $a_1 = 128, \quad a_{k+1} = \frac{1}{4}a_k$

$a_2 = \frac{1}{4}(128) = 32$

$a_3 = \frac{1}{4}(32) = 8$

$a_4 = \frac{1}{4}(8) = 2$

$a_5 = \frac{1}{4}(2) = \frac{1}{2}$

15. $a_1 = 3, \quad a_{k+1} = 1/a_k$

$a_2 = \frac{1}{3}$

$a_3 = \frac{1}{\frac{1}{3}} = 3$

$a_4 = \frac{1}{3}$

$a_5 = \frac{1}{\frac{1}{3}} = 3$

16. $a_1 = 2, \quad a_{k+1} = (a_k)^k$

$a_2 = (2)^1 = 2$

$a_3 = (2)^2 = 4$

$a_4 = (4)^3 = 4^3$

$a_5 = (4^3)^4 = 4^{12}$

17. Find the first four terms of the sequence of partial sums for the given sequence $\{3 + \frac{1}{2}n\}$.

$S_1 = a_1 = 3 + \frac{1}{2}(1) = \frac{7}{2}$

$S_2 = S_1 + a_2 = \frac{7}{2} + (3 + \frac{1}{2}(2)) = \frac{7}{2} + 4 = \frac{15}{2}$

$S_3 = S_2 + a_3 = \frac{15}{2} + (3 + \frac{1}{2}(3)) = \frac{15}{2} + \frac{9}{2} = 12$

$S_4 = S_3 + a_4 = 12 + (3 + \frac{1}{2}(4)) = 12 + 5 = 17$

Find the sum.

18. $\sum_{k=1}^{5}(2k-7)$

$= -5 + -3 + -1 + 1 + 3$

$= -5$

19. $\sum_{k=1}^{4}(k^2-5)$

$= (-4) + (-1) + 4 + 11$

$= 10$

20. $\sum_{k=3}^{6}\frac{k-5}{k-1}$

$= (-1) + \left(-\frac{1}{3}\right) + 0 + \frac{1}{5}$

$= -\frac{17}{15}$

21. $\sum_{k=1}^{5}(-3)^{k-1}$

$= 1 + -3 + 9 + -27 + 81$

$= 61$

22. $\sum_{k=1}^{1000} 5$

$= 1000(5)$

$= 5000$

23. $\sum_{k=253}^{571}\frac{1}{3}$

$= (571 - 253 + 1)\frac{1}{3}$

$= (319)\left(\frac{1}{3}\right)$

$= \frac{319}{3}$

24. $\sum_{j=1}^{7}\frac{1}{2}k^2$

$7\left(\frac{1}{2}k^2\right) =$

$\frac{7}{2}k^2$

Note: j, not k, is summation variable

25. The number of bacteria in a certain culture is initially 500, and the culture doubles in size every day.

(a) Find the number of bacteria present after one day, two days, and three days.

after 1 day, 1000
2 days, 2000
3 days, 4000

(b) Find a formula for the number of bacteria present after n days.

$a_n = 500(2)^n$
$= a_1 r^n$

26. The Fibonacci sequence is defined recursively by $a_1 = 1$, $a_2 = 1$, $a_{k+1} = a_k + a_{k-1}$ for $k \geq 2$.

(a) Find the first ten terms of the sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55