*AAT (IC/HW)-Day 1

Chapter 10: 10-2 Arithmetic Sequences

Name Key

Date: ______ Period: ______

Show that the given sequence is arithmetic, and find the common difference.

1. -6, -2, 2, ..., 4n-10,...

$d=4$ $a_n = a_1 + (n-1)d$

$= -6 + (n-1)4$

$= -6 + 4n - 4 = 4n - 10$

Find the fifth term, the tenth term, and the nth term of the arithmetic sequence.

2. 16, 13, 10, 7,...

$d = 13 - 16 = -3$

$a_n = 16 + (n-1)(-3)$

$a_n = -3n + 19$

$a_5 = -3(5) + 19 = 4$

$a_{10} = -3(10) + 19 = -11$

3. 3, 2.7, 2.4, 2.1, ...

$d = 2.7 - 3 = -.3$

$a_n = 3 + (n-1)(-.3)$

$a_n = -.3n + 3.3$

$a_5 = -.3(5) + 3.3 = 1.8$

$a_{10} = -.3(10) + 3.3 = .3$

4. -7, -3.9, -0.8, 2.3, ...

$d = -3.9 - (-7) = 3.1$

$a_n = -7 + (n-1)(3.1)$

$a_n = 3.1n - 10.1$

$a_5 = 3.1(5) - 10.1 = 5.4$

$a_{10} = 3.1(10) - 10.1 = 20.9$

Find the common difference for the arithmetic sequence with the specified term.

5. $a_2 = 21$, $a_6 = -11$

$a_6 = a_1 + (6-1)d$; $a_2 = a_1 + (2-1)d$

$a_6 = a_1 + 5d$ $\quad a_2 = a_1 + d$

$-11 = a_1 + 5d$ $\quad 21 = a_1 + d$

$-11 = \cancel{a_1} + 5d$

$-\ 21 = \cancel{a_1} + d$

$\frac{-32}{4} = \frac{4d}{4}$

$-8 = d$

Find the specified term of the arithmetic sequence that has the two given terms. $a_{15} = \frac{47}{17} + 14\left(\frac{36}{17}\right) = \frac{551}{17}$

6. a_{12}; $a_1 = 9.1$, $a_2 = 7.5$

$d = 7.5 - 9.1 = -1.6$

$a_{12} = 9.1 + (12-1)(-1.6)$

$a_{12} = -8.5$

7. a_1; $a_6 = 2.7$, $a_7 = 5.2$

$d = 5.2 - 2.7 = 2.5$

$a_6 = a_1 + (n-1)d$

$2.7 = a_1 + (6-1)2.5$

$2.7 = a_1 + 12.5$

$a_1 = -9.8$

8. a_{15}; $a_3 = 7$, $a_{20} = 43$

$7 = a_1 + (3-1)d$

$* \ 7 = a_1 + 2d$

$43 = a_1 + (20-1)d$

$* \ 43 = a_1 + 19d$

$7 = \cancel{a_1} + 2d$

$-\ 43 = \cancel{a_1} + 19d$

$-36 = -17d$ $\quad d = \frac{36}{17}$

$7 = a_1 + 2\left(\frac{36}{17}\right)$

$a_1 = \frac{47}{17}$

Find the sum S_n of the arithmetic sequence that satisfies the stated conditions.

9. $a_1 = 40, \quad d = -3, \quad n = 30$

$S_{30} = \frac{30}{2}\left[2(40) + 29(-3)\right]$

$= -105$

10. $a_1 = -9, \quad a_{10} = 15, \quad n = 10$

$S_{10} = \frac{10}{2}(-9 + 15)$

$= 30$

Find the sum.

11. $\sum_{k=1}^{20}(3k - 5)$

$a_1 = 3(1) - 5 = \underline{-2}$

$a_{20} = 3(20) - 5 = \underline{55}$

$S_{20} = \frac{20}{2}(-2 + 55)$

$= 530$

12. $\sum_{k=1}^{12}(7 - 4k)$

$a_1 = 7 - 4(1) = \underline{3}$

$a_{12} = 7 - 4(12) = \underline{-41}$

$S_{12} = \frac{12}{2}(3 + -41)$

$= -228$

13. $\sum_{k=1}^{18}(\frac{1}{2}k + 7)$

$a_1 = \frac{1}{2}(1) + 7 = \underline{\frac{15}{2}}$

$a_{18} = \frac{1}{2}(18) + 7 = \underline{16}$

$S_{18} = \frac{18}{2}\left(\frac{15}{2} + 16\right)$

$= \frac{423}{2}$

Express the sum in terms of summation notation.

14. $1 + 3 + 5 + 7$ $\quad d = 3 - 1 = 2$

$a_n = 1 + (n-1)2$

$= 1 + 2n - 2$

$= 2n - 1$

$\sum_{n=1}^{4}(2n - 1)$

15. $1 + 3 + 5 + \ldots + 73$

$2n - 1 = 73$

$2n = 74$

$n = 37$

$\sum_{n=1}^{37}(2n - 1)$

16. $\frac{3}{7} + \frac{6}{11} + \frac{9}{15} + \frac{12}{19} + \frac{15}{23} + \frac{18}{27}$ $\rightarrow +3$ $\rightarrow +4$

num: $3 + (n-1)3 = \underline{3n}$

den: $7 + (n-1)4 = \underline{4n+3}$

$\sum_{n=1}^{6}\frac{3n}{4n+3}$

17. Find the number of terms in the arithmetic sequence with $a_1 = -2$, $d = \frac{1}{4}$, $S = 21$.

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right) \quad \text{or} \quad S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$21 = \frac{n}{2}\left(2(-2) + (n-1)\tfrac{1}{4}\right)$$

$$42 = n\left(\tfrac{1}{4}n - \tfrac{17}{4}\right)$$

$$168 = n^2 - 17n$$

$$0 = n^2 - 17n - 168$$

$$0 = (n-24)(n+7)\ ; \quad n = 24 \quad \cancel{n = -7}$$

18. Insert five arithmetic means between 2 and 10.

$$2,\ \frac{10}{3},\ \frac{14}{3},\ \frac{18}{3} = 6,\ \frac{22}{3},\ \frac{26}{3},\ 10$$

$$a_7 = a_1 + (n-1)d$$

$$10 = 2 + 6d$$

$$d = \frac{4}{3}$$

19. A pile of logs has 24 logs in the bottom layer, 23 in the second layer, 22 in the third, and so on. The top layer contains 10 logs. Find the total number of logs in the pile.

$(24 - 10 + 1) = 15$ layers

$a_1 = 10$

$a_{15} = 24$

$$S_{15} = \frac{15}{2}(10 + 24) = 255 \text{ logs}$$

20. The first ten rows of seating in a certain section of a stadium have 30 seats, 32 seats, 34 seats, and so on. The eleventh through the twentieth rows each contain 50 seats. Find the total number of seats in on the section.

$a_1 = 30$

$d = 2$

$$S_{10} = \frac{10}{2}\left(2(30) + (10-1)(2)\right) = \underline{390}$$

Rows 11-20 each have

$10(50) = \underline{500}$ seats

total $= 390 + 500 = 890$ Seats

21. A contest will have five cash prizes totaling \$5000, and there will be a \$100 difference between successive prizes. Find the first prize.

$n = 5$

$S_5 = 5000$

$d = -100$

$$5000 = \frac{5}{2}\left(2a_1 + 4(-100)\right)$$

$$2000 = 2a_1 - 400$$

$$a_1 = \$1200$$