*AAT (IC/HW)-Day 1 Name Key

Chapter 10: 10-3 Geometric Sequences Date: ________ Period: ________

Show that the given sequence is geometric, and find the common ratio.

recursive

$a_{k+1} = a_k \cdot r$

explicit

$a_n = a_1 r^{n-1}$

1. $5, -\frac{5}{4}, \frac{5}{16}, \ldots, 5(-\frac{1}{4})^{n-1}, \ldots$

$r = \frac{-\frac{5}{4}}{5} = -\frac{1}{4}$

$a_n = a_1 r^{n-1}$
$= 5(-\frac{1}{4})^{n-1}$

Find the fifth term, the eighth term, and the nth term of the geometric sequence.

2. 8, 4, 2, 1, … $r = \frac{4}{8} = \frac{1}{2}$

$a_n = 8(\frac{1}{2})^{n-1}$ explicit

$a_5 = 8(\frac{1}{2})^{5-1} = \frac{8}{16} = \frac{1}{2}$

$a_8 = 8(\frac{1}{2})^{8-1} = \frac{1}{16}$

$a_n = a_1 r^{n-1}$

3. 300, -30, 3, -0.3, …

$r = \frac{-30}{300} = -\frac{1}{10}$

$a_n = 300(-\frac{1}{10})^{n-1}$

$a_5 = 300(-\frac{1}{10})^4 = \frac{3}{100} = .03$

$a_8 = 300(-\frac{1}{10})^7 = -0.00003$

4. 5, 25, 125, 625, …

$r = \frac{25}{5} = 5$

$a_n = 5(5)^{n-1}$

$a_5 = 5^5 = 3125$

$a_8 = 5^8 = 390,625$

5. 4, -6, 9, -13.5, …

$r = \frac{-6}{4} = -1.5$

$a_n = 4(-1.5)^{n-1}$

$a_5 = 4(-1.5)^4 = 20.25$

$a_8 = 4(-1.5)^7 = -68.34375$

6. $1, -x^2, x^4, -x^6, \ldots$

$r = \frac{-x^2}{1} = -x^2$

$a_n = 1(-x^2)^{n-1}$

$a_5 = x^8$

$a_8 = -x^{14}$

7. $2, 2^{x+1}, 2^{2x+1}, 2^{3x+1}, \ldots$

$r = \frac{2^{x+1}}{2^1} = 2^{x+1-1} = 2^x$

$a_n = 2(2^x)^{n-1}$

$a_5 = 2^{4x+1}$ $2 \cdot 2^{4x}$

$a_8 = 2^{7x+1}$

Find all possible values of r for a geometric sequence with the two given terms.

8. $a_4 = 3, \quad a_6 = 9$

$a_6 = a_1 r^5$; $a_4 = a_1 r^3$

$9 = a_1 r^5$ $\quad 3 = a_1 r^3$

$\frac{9}{r^5} = a_1$ $\quad \frac{3}{r^3} = a_1$

$\frac{9}{r^5} = \frac{3}{r^3}$

$9r^3 = 3r^5$

$3 = r^2$

$\pm\sqrt{3} = r$

– or –

$r^{6-4} = \frac{9}{3}$

$r^2 = 3$

$r = \pm\sqrt{3}$

9. Find the seventh term of the geometric sequence whose second and third terms are 2 and $-\sqrt{2}$.

$r = \frac{-\sqrt{2}}{2}$; $a_1 = \frac{a_2}{r} = \frac{-2}{\frac{-\sqrt{2}}{2}} = -2\sqrt{2}$

$a_7 = -2\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)^6 = \frac{-\sqrt{2}}{4}$

10. Given a geometric sequence with $a_4 = 4$ and $a_7 = 12$, find r and a_{10}.

$4 = a_1 r^3 \rightarrow \frac{4}{r^3} = a_1$

$12 = a_1 r^6 \rightarrow \frac{12}{r^6} = a_1$

$\frac{4}{r^3} = \frac{12}{r^6}$

$r^3 = 3$

$r = \sqrt[3]{3}$

$\frac{4}{(\sqrt[3]{3})^3} = \frac{4}{3} = a_1$

$a_n = \frac{4}{3}(\sqrt[3]{3})^{n-1} = \frac{4}{3}(\sqrt[3]{3})^{10-1} = 36$

$a_{10} = a_7 r^3$

$= 12(\sqrt[3]{3})^3$

$= 12(3)$

$= 36$

$\overline{a_7}\ \overline{a_8}\ \overline{a_9}\ \overline{a_{10}}$

$a_1 r^{n-1}$

a_7

Find the sum.

$a_1 = 3; a_2 = 9; r = \frac{9}{3} = 3$

11. $\sum_{k=1}^{10} 3^k$

$S_n = a_1 \cdot \frac{1-r^n}{1-r}$

$= 3 \cdot \frac{1-3^{10}}{1-3}$

$= 3 \cdot \frac{-59{,}048}{-2} = 88{,}572$

12. $\sum_{k=1}^{9}(-\sqrt{5})^k$

$((-\sqrt{5}))^{(+)}$

$= -\sqrt{5} \cdot \frac{1-(-\sqrt{5})^9}{1-(-\sqrt{5})}$

$= -\sqrt{5} \cdot \frac{(1+625\sqrt{5})}{1+\sqrt{5}} \cdot \frac{1-\sqrt{5}}{1-\sqrt{5}}$

$= \frac{3124\sqrt{5}-3120}{-4} = 780 - 781\sqrt{5}$

≈ -966.37

13. $\sum_{k=0}^{9}(-\frac{1}{2})^{k+1}$

rewrite -or-

$= \sum_{k=1}^{10}(-\frac{1}{2})^k = -\frac{1}{2} \cdot \frac{1-\frac{1}{2}^{10}}{1-(-\frac{1}{2})}$

$r = \frac{a_2}{a_1} = \frac{\frac{1}{4}}{-\frac{1}{2}} = \frac{1}{4} \cdot -2 = -\frac{1}{2}$

$1-(-\frac{1}{2})^{10}$

$= -\frac{1}{2} \cdot \frac{\frac{1023}{1024}}{\frac{3}{2}} \quad \frac{-1023}{2048} \div \frac{3}{2}$

$= \frac{-1023}{3072} = \frac{-341}{1024}$

Express the sum in terms of summation notation.

14. 2 + 4 + 8 + 16 + 32 + 64 + 128

$2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7$

$\sum_{n=1}^{7} 2^n$

-or-

$\sum_{n=1}^{7} a_1 r^{n-1} = 2^1 2^{n-1}$

$= 2^{1+n-1}$

$= 2^n$

15. $\frac{1}{4} - \frac{1}{12} + \frac{1}{36} - \frac{1}{108}$

$\frac{1}{4} - \frac{1}{4}\cdot\frac{1}{3^1} + \frac{1}{4}\cdot\frac{1}{3^2} - \frac{1}{4}\cdot\frac{1}{3^3}$

$= \sum_{n=1}^{4} \frac{1}{4}\left(-\frac{1}{3}\right)^{n-1}$

Find the sum of the infinite geometric series if it exists.

if $|r| < 1$

$S = \frac{a_1}{1-r}$

16. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

$a_1 = 1$

$r = -\frac{1}{2}$

$S = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$

17. 1.5 + 0.015 + 0.00015 + ...

$a_1 = 1.5$

$r = .01$

$S = \frac{1.5}{1-.01} = \frac{50}{33}$

18. 256 + 192 + 144 + 108 + ...

$a_1 = 256$

$r = \frac{192}{256} = \frac{3}{4}$

$S = \frac{256}{1-\frac{3}{4}} = 1024$

19. Find the geometric mean of 12 and 48.

$$\frac{x}{12} = \frac{48}{x}$$

$$x^2 = 576$$

$$x = 24$$

12, $\underline{24}$, 48

geo mean of a : b = $\sqrt{ab}$

20. Insert two geometric means between 4 and 500.

4, $\underline{20}$, $\underline{100}$, 500

$$a_1 r^{n-1} = a_n$$

$$4r^3 = 500$$

$$r^3 = 125$$

$$\underline{r = 5}$$

21. If a deposit of \$100 is made on the first day of each month into an account that pays 6% interest per year compounded monthly, determine the amount in the account after 18 years.

$P\left(1 + \frac{R}{n}\right)^{K}$ → # of compounding periods for each deposit → $K = 18 \cdot 12 = \underline{216}$

$$a_K = \underbrace{100\left(1 + \frac{.06}{12}\right)}_{r}^{K} = 100(1.005)^K \quad S_n = a_1 \frac{(1-r^n)}{(1-r)} = \quad S_{216} = a_1 + a_2 + \ldots a_{216}$$

ratio

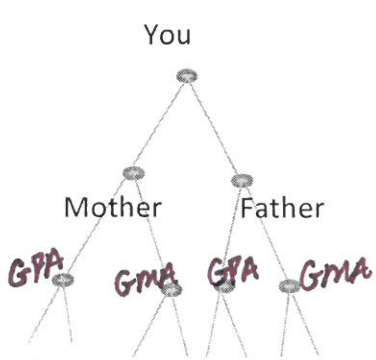
$$= 100(1.005) + 100(1.005)^2 + \ldots 100(1.005)^{216}$$

$$= \underbrace{100(1.005)}_{a_1}^{K}$$

a_1

$$= 100(1.005) \cdot \frac{(1-(1.005)^{216})}{(1-(1.005))} \approx \$38,929.00$$

22. Shown in the figure is a family tree displaying the current generation (you) and 3 prior generations, with a total of 12 grandparents. If you were to trace your family history back 10 generations, how many grandparents would you find?



2 generations = 4 grandparents = 2^2

3 generations = 8 grandparents = 2^3

n generations = 2^n " "

$$4 + 8 = 12$$

$$\sum_{K=2}^{10} 2^K = \sum_{K=1}^{9} 2^{K+1}$$

$$S_n = a_1 \frac{1-r^n}{1-r} = 4 \cdot \frac{(1-2^9)}{1-2} = 2044$$

10 generations $= 2^{10} = 1024$