

Find the number.

1.  $P(7, 3)$

$$= \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{4!} = 210$$

2.  $P(9, 6) = \frac{9!}{3!}$

$$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{3!} = 60,480$$

3.  $P(5, 5) = \frac{5!}{0!}$

$$= \frac{5!}{1} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

4.  $P(6, 1)$

$$= \frac{6!}{5!} = \frac{6 \cdot \cancel{5!}}{5!} = 6$$

Simplify the permutations.

5.  $P(n, 0) = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

6.  $P(n, n-1) = \frac{n!}{(n-(n-1))!} = \frac{n!}{1} = n!$

7.  $P(n, 1) = \frac{n!}{(n-1)!} = \frac{n \cdot \cancel{(n-1)!}}{\cancel{(n-1)!}} = n$

8. How many three-digit numbers can be formed from the digits 1,2,3,4, and 5 if repetitions

(a) are not allowed

$$\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$$

(b) are allowed

$$\underline{5} \cdot \underline{5} \cdot \underline{5} = 125$$

9. In a certain state, automobile license plates start with one letter of the alphabet, followed by five digits (0,1,2,...9). Find how many different license plates are possible if

(a) the first digit following the letter cannot be 0

$$\underline{26} \cdot \underline{9} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 26 \cdot 9 \cdot 10^4 = 2,340,000$$

(b) the first letter cannot be O or I and the first digit cannot be 0

$$\underline{24} \cdot \underline{9} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 2,160,000$$

10. A row of six seats in a classroom is to be filled by selecting individuals from a group of ten students.

(a) In how many different ways can the seats be occupied?

$$P(10,6) = \frac{10!}{4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{4!} = 151,200$$

(b) If there are six boys and four girls in the group and if boys and girls are to be alternated, find the number of different seating arrangements.

$$\text{Boy-girl: } \boxed{6} \cdot \boxed{4} \cdot \boxed{5} \cdot \boxed{3} \cdot \boxed{4} \cdot \boxed{2} = 2880$$

$$\text{Girl-boy: } \boxed{4} \cdot \boxed{6} \cdot \boxed{3} \cdot \boxed{5} \cdot \boxed{2} \cdot \boxed{4} = 2880$$

$$2880 + 2880 = 5760$$

11. In how many different ways can a test consisting of ten true-or-false questions be completed?

$$\underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2} \cdot \underline{2}$$

$$2^{10} = 1024$$

12. In how many different ways can eight people be seated in a row?

$$P(8,8) = \frac{8!}{0!} = 8! = 40,320$$

13. With six different flags, how many different signals can be sent by placing three flags, one above the other, on a flag pole?

$$P(6,3) = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{3!} = 120$$

14. There are 24 letters in the Greek alphabet. How many fraternities may be specified by choosing three Greek letters if repetitions

(a) are not allowed  $P(24,3) = \frac{24!}{21!} = 24 \cdot 23 \cdot 22 = 12,144$

(b) are allowed  $\underline{24} \cdot \underline{24} \cdot \underline{24} = 13,824$

15. How many seven-digit phone numbers can be formed from the digits 0,1,2,3,...9 if the first digit may not be 0?

$$9 \cdot 10^6 = 9,000,000$$

16. A customer remembers that 2, 4, 7, and 9 are the digits of a four-digit access code for an automatic bank-teller machine. Unfortunately, the customer has forgotten the order of the digits. Find the largest possible number of trials necessary to obtain the correct code.

$$P(4, 4) = \frac{4!}{0!} = 4! = 24$$

17. Ten horses are entered in a race. If the possibility of a tie for any place is ignored, in how many ways can the first-, second-, and third-place winners be determined?

$$P(10, 3) = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720$$