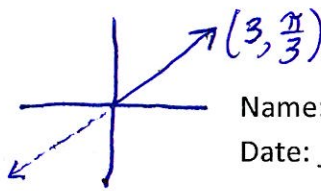


*AAT

Chapter 11-5: Polar Coordinates (IC/HW)



Name: Key

Date: _____ Period: _____

Which polar coordinates represent the same point as $(3, \pi/3)$?

1. (a) $(3, 7\pi/3)$ (b) $(3, -\pi/3)$ (c) $(-3, 4\pi/3)$ (d) $(3, -2\pi/3)$ (e) $(-3, -2\pi/3)$ (f) $(-3, -\pi/3)$

Change the polar coordinates to rectangular coordinates.

2. $(3, \pi/4)$

$$x = r \cos \theta = 3 \cos \frac{\pi}{4} = 3 \left(\frac{\sqrt{2}}{2} \right) = \frac{3\sqrt{2}}{2}$$

$$y = r \sin \theta = 3 \sin \frac{\pi}{4} = 3 \left(\frac{\sqrt{2}}{2} \right) = \frac{3\sqrt{2}}{2}$$

3. $(-1, 2\pi/3)$

$$x = -1 \cos \frac{2\pi}{3} = -1 \left(-\frac{1}{2} \right) = \frac{1}{2}$$

$$y = -1 \sin \frac{2\pi}{3} = -1 \left(\frac{\sqrt{3}}{2} \right) = -\frac{\sqrt{3}}{2}$$

4. $(8, -2\pi/3)$

$$x = 8 \cos \left(-\frac{2\pi}{3} \right) = 8 \left(-\frac{1}{2} \right) = -4$$

$$y = 8 \sin \left(-\frac{2\pi}{3} \right) = 8 \left(-\frac{\sqrt{3}}{2} \right) = -4\sqrt{3}$$

5. $(-3, 5\pi/3)$

$$x = -3 \cos \frac{5\pi}{3} = -3 \left(\frac{1}{2} \right) = -\frac{3}{2}$$

$$y = -3 \sin \frac{5\pi}{3} = -3 \left(-\frac{\sqrt{3}}{2} \right) = \frac{3\sqrt{3}}{2}$$

Change the rectangular coordinates to polar coordinates with $r > 0$ and $0 \leq \theta < 2\pi$.

6. $(-1, 1)$

$$r^2 = x^2 + y^2 = (-1)^2 + (1)^2 = 2$$

$$r = \sqrt{2}, \tan \theta = \frac{y}{x} = \frac{1}{-1} = -1$$

$$\theta = \tan^{-1}(-1) = \frac{3\pi}{4}$$

7. $(-2\sqrt{3}, -2)$

$$r^2 = (-2\sqrt{3})^2 + (-2)^2 = 12 + 4 = 16$$

$$r = 4, \tan \theta = \frac{-2}{-2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{\pi}{6}$$

8. $(7, -7\sqrt{3})$

$$r^2 = 7^2 + (-7\sqrt{3})^2 = 49 + 147 = 196$$

$$r = 14, \tan \theta = \frac{-7\sqrt{3}}{7} = -\sqrt{3}$$

$$\theta = \tan^{-1}(-\sqrt{3}) = \frac{5\pi}{3}$$

9. $(5, 5)$

$$r^2 = 5^2 + 5^2 = 50$$

$$r = 5\sqrt{2}, \tan \theta = \frac{5}{5} = 1$$

$$\theta = \tan^{-1}(1) = \frac{\pi}{4}$$

Find a polar equation that has the same graph as the equation in x and y .

10. $x = -3$

$$r \cos \theta = -3$$

$$r = \frac{-3}{\cos \theta}$$

$$r = -3 \sec \theta$$

11. $x^2 + y^2 = 16$

$$r^2 = 16$$

Circle w/ radius = 4

12. $y^2 = 6x$

$$r^2 \sin^2 \theta = 6r \cos \theta$$

$$r \sin^2 \theta = 6 \cos \theta$$

$$r = \frac{6 \cos \theta}{\sin^2 \theta} = 6 \cot \theta \cdot \csc \theta$$

13. $x + y = 3$

$$r \cos \theta + r \sin \theta = 3$$

$$r (\cos \theta + \sin \theta) = 3$$

$$r = \frac{3}{\cos \theta + \sin \theta}$$

14. $2y = -x$

$$\frac{y}{x} = -\frac{1}{2}$$

$$\frac{r \sin \theta}{r \cos \theta} = -\frac{1}{2}$$

$$\tan \theta = -\frac{1}{2}$$

$$\theta = \tan^{-1} \left(-\frac{1}{2} \right)$$

15. $y^2 - x^2 = 4$

$$r^2 \sin^2 \theta - r^2 \cos^2 \theta = 4$$

$$-r^2 (\cos^2 \theta - \sin^2 \theta) = 4$$

$$-r^2 \cos 2\theta = 4$$

$$r^2 = \frac{-4}{\cos 2\theta}; r = -2 \sec 2\theta$$

16. $(x-1)^2 + y^2 = 1$

$$x^2 - 2x + 1 + y^2 = 1$$

$$x^2 + y^2 = 2x$$

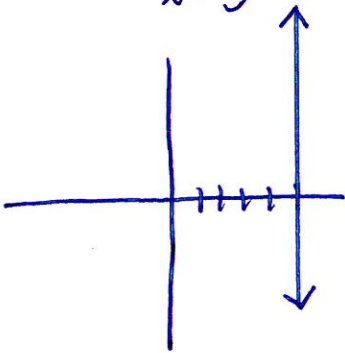
$$\frac{y^2}{r} = \frac{2r \cos \theta}{r}$$

$$r = 2 \cos \theta$$

Find an equation in x and y that has the same graph as the polar equation. Use it to sketch the graph in an $r\theta$ -plane.

1. $r \cos \theta = 5$

$x = 5$



2. $r - 6 \sin \theta = 0$

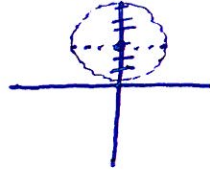
$r (r = 6 \sin \theta) r$

$r^2 = 6r \sin \theta$

$x^2 + y^2 = 6y$

$x^2 + y^2 - 6y + 9 = 9$

$x^2 + (y - 3)^2 = 9$

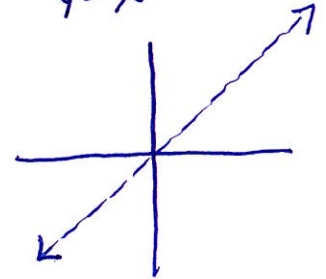


3. $\theta = \pi/4$

$\tan \theta = \tan \frac{\pi}{4}$

$\frac{y}{x} = 1$

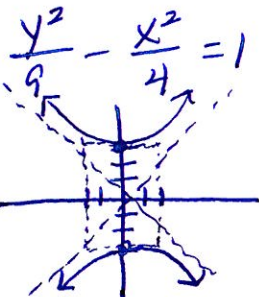
$y = x$



4. $r^2(4 \sin^2 \theta - 9 \cos^2 \theta) = 36$

$4r^2 \sin^2 \theta - 9r^2 \cos^2 \theta = 36$

$\frac{4y^2}{36} - \frac{9x^2}{36} = \frac{36}{36}$

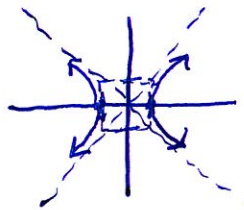


5. $r^2 \cos 2\theta = 1$

$r^2(\cos^2 \theta - \sin^2 \theta) = 1$

$r^2 \cos^2 \theta - r^2 \sin^2 \theta = 1$

$x^2 - y^2 = 1$

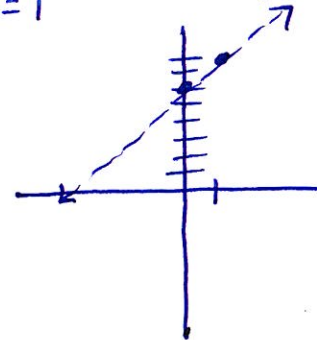


6. $r(\sin \theta - 2 \cos \theta) = 6$

$r \sin \theta - 2r \cos \theta = 6$

$y - 2x = 6$

$y = 2x + 6$

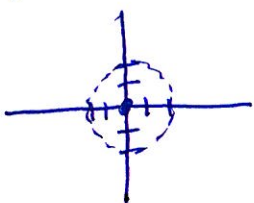


Sketch the graph of the polar equation.

7. $r = -2$

$r^2 = 4$

$x^2 + y^2 = 4$



8. $\theta = -\pi/6$

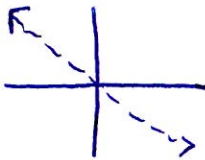
$\tan \theta = \frac{y}{x}$

$x \tan \theta = y$

$x \tan \frac{-\pi}{6} = y$

$x \left(-\frac{\sqrt{3}}{3} \right) = y$

$-\frac{\sqrt{3}}{3} x = y$



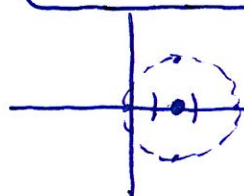
9. $r = 3 \cos \theta$

$r^2 = 3r \cos \theta$

$x^2 + y^2 = 3x$

$x^2 - 3x + \frac{9}{4} + y^2 = \frac{9}{4}$

$\left(x - \frac{3}{2} \right)^2 + y^2 = \frac{9}{4}$



10. $r = 4 \cos \theta + 2 \sin \theta$

$r^2 = 4r \cos \theta + 2r \sin \theta$

$x^2 + y^2 = 4x + 2y$

$x^2 - 4x + 4 + y^2 - 2y + 1 = 4 + 1$

$(x - 2)^2 + (y - 1)^2 = 5$

