

Chapter 10: Arithmetic & Geometric Sequences and Series (IC)

1. For each sequence, identify the next two terms. Write an explicit and recursive rule to generate the sequence.

a. 4, 9, 14, 19, ... 24, 29

arithmetic
recursive

$$a_{n+1} = a_n + 5$$

explicit

$$a_n = a_1 + (n-1)d$$

$$= 4 + (n-1)5$$

$$= 4 + 5n - 5$$

b. 2, 8, 32, 128, ... 512, 2048

geometric
recursive

$$a_{n+1} = a_n \cdot 4$$

$$a_n = 5n - 1$$

explicit
$$a_n = a_1 r^{n-1}$$

$$a_n = 2 \cdot 4^{n-1}$$

c. 1, 1, 2, 3, 5, ... (Hint: This is a special sequence...can you name it and write a recursive rule?)

Fibonacci Sequence

$$a_{n+1} = a_n + a_{n-1}$$

2. Determine if the following sequences are arithmetic or geometric. Write an explicit and recursive formula for each.

a. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

geometric

$$a_{k+1} = a_k \cdot \frac{1}{2} : \underline{\underline{R}}$$

$$a_n = a_1 r^{n-1}$$

$$1 \cdot \frac{1}{2}^{n-1} : \underline{\underline{E}}$$

b. -7, -4, -1, 2, ...

arithmetic

$$a_n = -7 + 3(n-1)$$

$$= -7 + 3n - 3$$

$$a_n = -10 + 3n : \underline{\underline{E}}$$

$$a_{n+1} = a_n + 3 : \underline{\underline{R}}$$

arithmetic

c. 100, 87, 74, 61, ...

$$a_n = 100 - 13(n-1)$$

$$= 100 - 13n + 13$$

$$a_n = 113 - 13n : \underline{\underline{E}}$$

$$a_{k+1} = a_k - 13 : \underline{\underline{R}}$$

3. For each sequence, write an explicit formula and then calculate the specified sum.

a. 2, 9, 16, 23, ...; sum of the first 18 terms

$$a_n = 2 + 7(n-1)$$

$$= 2 + 7n - 7$$

$$a_n = 7n - 5$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{18} = \frac{18}{2}(2 + 121) = 1107$$

b. -6, -2, 2, 6, ...; sum of the first 21 terms

$$a_n = -6 + 4(n-1)$$

$$= -6 + 4n - 4$$

$$a_n = 4n - 10$$

$$S_{21} = \frac{21}{2}(-6 + 74) = 714$$

c. 45, 36, 27, 18, ...; sum of the first 11 terms

$$a_n = 45 - 9(n-1)$$

$$= 45 - 9n + 9$$

$$a_n = -9n + 54$$

$$S_{11} = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{11}{2}(45 + -45)$$

$$= \frac{11}{2}(0)$$

$$= 0$$