Openers #10

Name: Key

Each day when you come into class, there will be a problem projected for you to complete. Find the appropriate box to complete the problem in and work on it when you arrive.

Date: ____/____/____

10-1

Find the first four terms and the ~~eighth term of the sequence~~. {(n-1)(n-2)(n-3)}

0, 0, 0, 6; 210

Find the first five terms of the recursively defined infinite sequence. $a_1=2$, $a_{k+1} = (a_k)^{1/k}$

$2, 2, 2^{1/2}, 2^{1/6}, 2^{1/24}$

a_1 a_2

Find the sum. $\sum_{k=0}^{5} k(k-2)$

$0+(-1)+0+3+8+15 = 25$

Find the sum. $\sum_{k=1}^{100} 100$ $= 100(100) = 10{,}000$

Date: ____/____/____

10-2

Find the fifth term, the tenth term, and the nth term of the arithmetic sequence. 2,6,10,14,...

$d = 6-2 = 4$; $a_n = 2+(n-1)(4)$ $a_5 = 18$

$= 4n-2$ $a_{10} = 38$

Find the common difference for the arithmetic sequence with $a_4 = 14$, $a_{11} = 35$.

-or-

$\frac{35-14}{7} = 3 = d$

$a_{11} = a_1 + 10d \rightarrow 35 = a_1 + 10d$

$a_4 = a_1 + 3d \rightarrow 14 = a_1 + 3d$

$21 = 7d$; $d = 3$

Find the specified term of the arithmetic sequence that ha the two given terms.

a_{11}; $a_1 = 2+\sqrt{2}$; $a_2 = 3$.

$d = 3-(2+\sqrt{2})$

$d = 1-\sqrt{2}$;

$a_{11} = (2+\sqrt{2}) + 10(1-\sqrt{2})$

$= 12-9\sqrt{2} \approx -.7279$

Find the sum. $\sum_{k=1}^{10} \frac{1}{4}k + 3$

$a_1 = \frac{13}{4}$

$a_{10} = \frac{11}{2}$

$S_{10} = \frac{10}{2}\left(\frac{13}{4} + \frac{11}{2}\right) = \frac{175}{4} \approx 43.75$

Date: ___/___/___

10-3

Find the fifth term, the eighth term, and the nth term of the geometric sequence. 2,6,18,54,...

$r = \frac{6}{2} = 3$

$a_n = 2(3)^{n-1}$

$a_n = a_1 r^{n-1}$

$a_5 = 2 \cdot 3^4 = 162$

$a_8 = 2 \cdot 3^7 = 4374$

Given a geometric sequence with $a_2 = 3$ and $a_5 = -81$, find r and a_9.

$3 = a_1 r^1$

$-81 = a_1 r^4$

$a_1 = \frac{3}{r}$; $a_1 = \frac{-81}{r^4}$

$\frac{-81}{r^4} = \frac{3}{r}$

$-81r = 3r^4$

$-81 = 3r^3$

$-27 = r^3$

$-3 = r$

$r^3 = \frac{-81}{3} = -27$

$r = -3$

$a_1 = \frac{3}{-3} = -1$

$a_9 = a_1 r^{n-1}$

$a_9 = (-1)(-3)^8$

$a_9 = -6561$

Find the sum. $\sum_{k=1}^{7} 3^{-k} = \left(\frac{1}{3}\right)^k$

$S_n = a_1 \frac{1-r^n}{1-r}$

$\frac{1}{3} \frac{1-\frac{1}{3}^7}{(1-\frac{1}{3})} = \frac{1}{3} \frac{\left(\frac{2186}{2187}\right)}{\frac{2}{3}} = \frac{1093}{2187} \approx .499$

Find the sum of the infinite geometric series if it exists. 250 – 100 + 40 – 16 + ...

$a_1 = 250$

$r = -\frac{2}{5}$

$S = \frac{250}{1+\frac{2}{5}} = \frac{1250}{7}$

$S = \frac{a_1}{1-r}$

Date: ___/___/___

10-5

Evaluate the expression. $\binom{52}{2} \frac{52!}{2!(52-2)!} = \frac{52!}{2!50!} = \frac{52 \cdot 51 \cdot 50!}{2 \cdot 50!} = 1326$

Rewrite the expression so it does not contain factorials. $\frac{(n+1)!}{(n-1)!} \frac{(n+1)(n)(n-1)!}{(n-1)!} = n(n+1)$

Use Pascal's Triangle to simplify. $(a-b)^5$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

Date:

___/___/___

10-6

Find the number. P(5,1) $\frac{5!}{4!} = 5$

Simplify the permutation. P(n,2) $\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!} = n(n-1)$

How many four digit numbers can be formed from the digits 1,2,3,4,5,6 if repetitions are

a) not allowed. $P(6,4) = 360$ $6 \cdot 5 \cdot 4 \cdot 3$

b) allowed $6^4 = 1296$ $6 \cdot 6 \cdot 6 \cdot 6$

In how many ways can 13 cards be selected from a deck of cards?

$P(52,13) = 3.954 \times 10^{21}$

$52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46 \times 45 \times 44 \times 43 \times 42 \times 41 \times 40$

In how many ways can 13 cards be selected to obtain 5 spades, 3 hearts, 3 clubs, and 2 diamonds?

$P(13,5) \cdot P(13,3) \cdot P(13,3) \cdot P(13,2) = 7.054 \times 10^{13}$

$(13 \times 12 \times 11 \times 10 \times 9) \times (13 \times 12 \times 11) \times (13 \times 12 \times 11) \times (13 \times 12)$

Date:

___/___/___

10-7

Find the number. C(6,2) $\frac{6!}{4!2!} = 15$

Find the number of distinguishable permutations of the letters in the word *bookkeeper*.

$$\frac{10!}{3!2!2!1!1!1!} = 151,200$$

a) If a student must answer 8 of 12 questions on an examination, how many different selections of questions are possible?

$C(12,8) = 495$

b) How many selections are possible if the first three questions must be answered?

$C(9,5) = 126$

10-8

A single card is drawn from a deck. Find the probability that the card is

a) a heart $13/52 = (1/4)$
b) a heart or a diamond $26/52 = (\frac{1}{2})$ $13/52 + 13/52$
c) a heart, diamond or a club $39/52 = (3/4)$ $13/52 + 13/52 + 13/52 = 39/52$

A single die is tossed. Find the probability and the odds that the die is

a) an even number $3/6$; O(E) 3 to 3 or 1 to 1
b) a number divisible by 5 $1/6$; O(E) 1 to 5
c) an even number or a number divisible by 5

$\frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$; O(E) = 4 to 2
or 2 to 1

A single card is drawn from a deck. Find the probability that the card is either red or a face card.

$$\frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}$$

$$P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

If 6 black, 5 red, 4 white, and 2 green disks are to be arranged in a row, what is the number of possible color arrangements.?

$$\frac{17!}{6!\,5!\,4!\,2!} = 85,765,680$$