Openers #11

Name: Key

Each day when you come into class, there will be a problem projected for you to complete. Find the appropriate box to complete the problem in and work on it when you arrive.

Date: ___/___/___

11-1

Find the vertex, focus, and directrix of the parabola. Sketch its graph, showing the focus and the directrix.

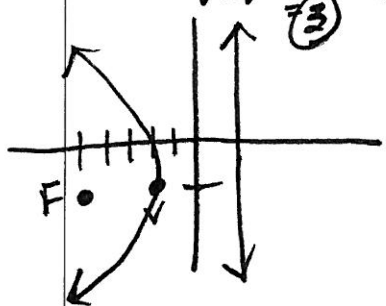
a) $(y+1)^2 = -12(x+2)$

$-\frac{1}{12}(y+1)^2 = x+2$

$-\frac{1}{12}(y+1)^2 - 2 = x$

$|p = \frac{1}{4a}|$ $V(-2,-1)$; $F(-5,-1)$

$p = \frac{1}{4(-\frac{1}{12})} = |-3| = 3$ directrix: $x = 1$



b) $y^2 - 4y - 2x - 4 = 0$

$y^2 - 4y + 4 = 4 + 4 + 2x$

$(y-2)^2 = 2x + 8$

$(y-2)^2 = 2(x+4)$

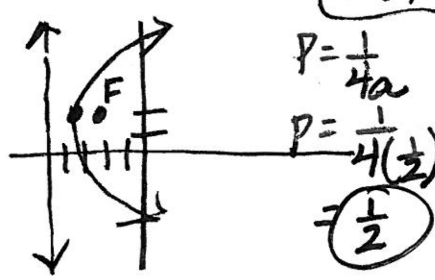
$\frac{1}{2}(y-2)^2 - 4 = x$

$V(-4,2)$ $F(-\frac{7}{2},2)$ 3.5

directrix $x = -\frac{9}{2}$ -4.5

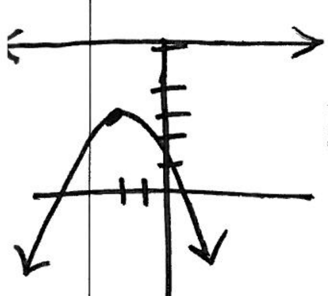
$p = \frac{1}{4a}$

$p = \frac{1}{4(\frac{1}{2})} = \frac{1}{2}$



Find an equation of the parabola that satisfies the given conditions.

a) Vertex V(-2,3); directrix **y** = 5



$p = 2$;

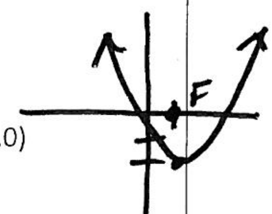
$a(x-h)^2 + k = y$

$-\frac{1}{8}(x+2)^2 + 3 = y$

$p = \frac{1}{4a}$

$2 = \frac{1}{4a}$; $\frac{8a}{8} = \frac{1}{8}$; $a = \frac{1}{8}$ (negative b/c opens down)

b) Vertex V(1,-2); focus F(1,0)



$p = 2$;

$a(x-h)^2 + k = y$

$\frac{1}{8}(x-1)^2 - 2 = y$

$p = \frac{1}{4a}$

$2 = \frac{1}{4a}$

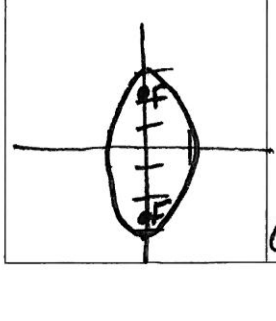
$8a = 1$; $a = \frac{1}{8}$ (positive b/c opens up)

Date: ___/___/___

11-2

Find the vertices and foci of the ellipse. Sketch the ellipse, showing the foci.

a) $y^2 + 9x^2 = 9$ $\frac{y^2}{9} + \frac{x^2}{1} = 1$



$c^2 = 9 - 1$

$c^2 = 8$

$c = \pm 2\sqrt{2}$

$V(0, \pm 3)$

$F(0, \pm 2\sqrt{2})$

$CV(\pm 1, 0)$

b) $x^2 + 2y^2 + 2x - 20y + 43 = 0$

$(x^2 + 2x + 1) + 2(y^2 - 10y + 25) = -43 + 1 + 50$

$(x+1)^2 + 2(y-5)^2 = 8$

$\frac{(x+1)^2}{8} + \frac{(y-5)^2}{4} = 1$

$c^2 = 8 - 4$

$c^2 = 4$

$c = \pm 2$

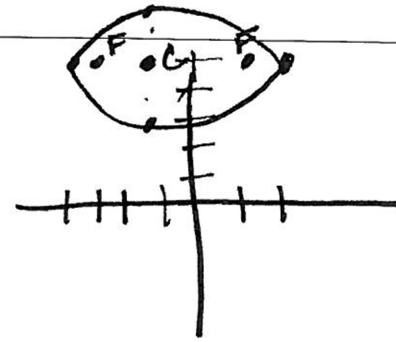
$C(-1,5)$

$V(-1 \pm 2\sqrt{2}, 5)$

$F(-1 \pm 2, 5)$

$CV(-1, 5 \pm 2)$

11-2 (continued…)



Find an equation for the ellipse that has its center at the origin and satisfies the given conditions.

a) Vertices V(0, ±7) ; foci F(0, ±2)

$b^2 = 7^2 - 2^2 = 45$

$\frac{x^2}{45} + \frac{y^2}{49} = 1$

b) Foci F(±3,0) ; minor axis of length 2

$2 = 2b$

$b = 1$

$a^2 = 3^2 + 1^2$

$a^2 = 10$

$\frac{x^2}{10} + \frac{y^2}{1} = 1$

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11-3

Find the vertices, the foci, and the equations of the asymptotes of the hyperbola. Sketch its graph, showing the asymptotes and the foci.

a) $X^2 - 2y^2 = 8$

$\frac{x^2}{8} - \frac{y^2}{4} = 1$

$V(\pm 2\sqrt{2}, 0)$

$F(\pm 2\sqrt{3}, 0)$

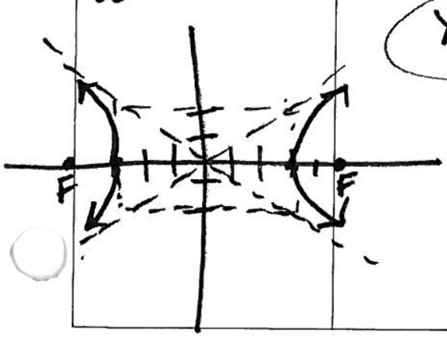
$CV(0, \pm 2)$

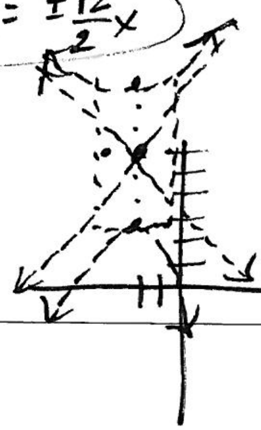
$c^2 = 8 + 4$

$c = \pm 2\sqrt{3}$

$y = \pm \frac{b}{a} x$

$y = \pm \frac{2}{2\sqrt{2}} = \pm \frac{\sqrt{2}}{2} x$





b) $y^2 - 4x^2 - 12y - 16x + 16 = 0$

$(y^2 - 12y + \underline{36}) - 4(x^2 + 4x + \underline{4}) = -16 + \underline{36} \; \underline{-16}$

$(y-6)^2 - 4(x+2)^2 = 4$

$\frac{(y-6)^2}{4} - \frac{(x+2)^2}{1} = 1$

$c^2 = 4 + 1 ; \; c = \pm\sqrt{5}$

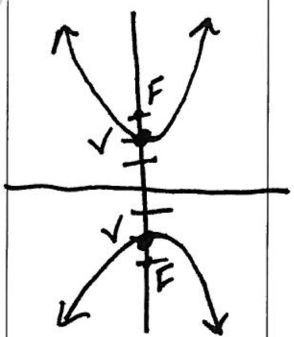
$C(-2, 6) ; \; V(-2, 6 \pm 2) ; \; F(-2, 6 \pm \sqrt{5})$

$CV(-2 \pm 1, 6) ; \; y = \pm \frac{a}{b} x$

$y = \pm \frac{2}{1} x$

11-3 (continued...)

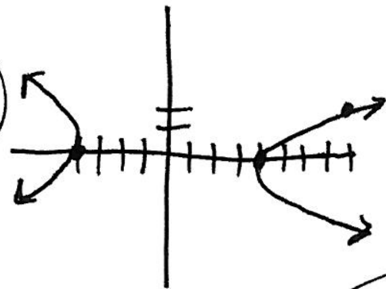
Find an equation for the hyperbola that has its center at the origin and satisfies the given conditions.



a) Foci F(0, ±3); vertices V(0, ±2)

$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$

b) Vertices V(±4,0); passing through (8,2)



$$\frac{x^2}{16} - \frac{y^2}{b^2} = 1$$

$$\frac{64}{16} - \frac{4}{b^2} = 1 \to 4 - \frac{4}{b^2} = 1$$

$$b^2 = \frac{4}{3}$$

$$\frac{x^2}{16} - \frac{y^2}{\frac{4}{3}} = 1$$

-or-

$$\frac{x^2}{16} - \frac{3y^2}{4} = 1$$

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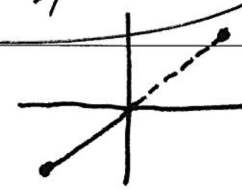
11-5



Change the polar coordinates to rectangular coordinates.

a) (4, -π/4)

$$x = 4\cos\frac{-\pi}{4} = 4\left(\frac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

$$y = 4\sin\frac{-\pi}{4} = 4\left(\frac{-\sqrt{2}}{2}\right) = -2\sqrt{2}$$

b) (-2, 7π/6)

$$x = -2\cos\frac{7\pi}{6} = -2\left(-\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

$$y = -2\sin\frac{7\pi}{6} = -2\left(-\frac{1}{2}\right) = 1$$

Change the rectangular coordinates to polar coordinates with r>0 and 0≤θ≤2π.

a) (-2√2, -2√2)

$$r^2 = (-2\sqrt{2})^2 + (-2\sqrt{2})^2 = 16$$

$$r = 4$$

$$\tan\theta = \frac{-2\sqrt{2}}{-2\sqrt{2}} = 1;\ \theta = \frac{5\pi}{4}\ (QIII)$$

$$\left(4, \frac{5\pi}{4}\right)$$

b) (-4, 4√3)

$$r^2 = (-4)^2 + (4\sqrt{3})^2 = 64$$

$$r = 8$$

$$\tan\theta = \frac{4\sqrt{3}}{-4} = -\sqrt{3}$$

$$\theta = \frac{2\pi}{3} \quad \left(8, \frac{2\pi}{3}\right)$$

Find a polar equation that has the same graph as the equation in x and y.

a) $x^2 + y^2 = 2$

$$r^2 = 2$$

$$r = \pm\sqrt{2}$$

Circle w/ radius √2

11-5 (continued…)

b) $x^2 = 8y$

$r^2\cos^2\theta = 8r\sin\theta$

$r^2\cos^2\theta - 8r\sin\theta = 0$

$\frac{r(r\cos^2\theta - 8\sin\theta)}{r} = \frac{0}{r}$

$r\cos^2\theta = 8\sin\theta$

$r = \frac{8\sin\theta}{\cos^2\theta} = 8\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$

$r = 8\tan\theta \cdot \sec\theta$

Find an equation in x and y that has the same graph as the polar equation.

a) r=2

$r^2 = 4$

$x^2 + y^2 = 4$

Circle $c(0,0)$
$r = 2$

b) $r^2(\cos^2\theta + 4\sin^2\theta) = 16$

$r^2\cos^2\theta + 4r^2\sin^2\theta = 16$

$x^2 + 4y^2 = 16$

$\frac{x^2}{16} + \frac{4y^2}{16} = 1;$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

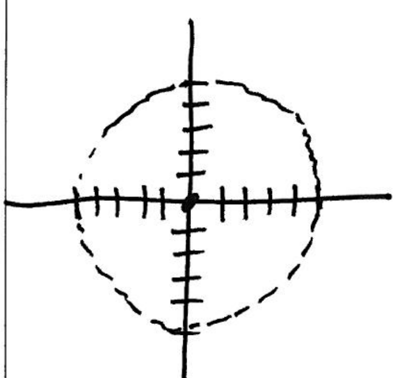
Sketch the graph of the polar equation.

a) r=5

$r^2 = 5^2$

$x^2 + y^2 = 25$

Circle
$c(0,0);\ r=5$



b) $r = -2\sin\theta$

$r^2 = -2r\sin\theta$

$x^2 + y^2 = -2y$

$x^2 + (y^2 + 2y + \underline{1}) = 0 + \underline{1}$

$x^2 + (y+1)^2 = 1$

Circle $(0,-1)$
$r = 1$