*AAT

Chapter 10: Review for Quiz 10.1-10.5, omit 10.4 (IC/HW)

Name: Key
Date: ____________ Period: ____________

1. Find the seventh term of the sequence.

$\left\{5+\frac{1}{n}\right\}$

$5+\frac{1}{7}=\frac{35}{7}+\frac{1}{7}=\frac{36}{7}$

2. Find the fourth term of the recursively defined infinite sequence.

$a_1 = 185,\ a_{k+1}=\frac{1}{4}a_k$

$a_1 = 185$

$a_2=\frac{1}{4}(185)=\frac{185}{4}$

$a_3=\frac{1}{4}\left(\frac{185}{4}\right)=\frac{185}{16}$

$a_4=\frac{1}{4}\left(\frac{185}{16}\right)=\frac{185}{64}=2\frac{57}{64}$

3. Consider the sequence defined recursively by $a_{k+1}=\sqrt{a_k}$, $a_1 = 9$. What number does this sequence approach as k increases?

$a_1 = 9$

$a_2=\sqrt{9}=3$

$a_3=\sqrt{3}\approx 1.7\ldots$

$a_4=\sqrt{1.7}\approx 1.3\ldots$

$a_5=\sqrt{1.3}=1.14\ldots$

$\boxed{L=1}$

4. The number of bacteria in a certain culture is 500, and the culture doubles in size every day. Find the number of bacteria present after three days.

0 — initial = 500

1 — day 1 = 1000

2 — day 2 = 2000

3 — day 3 = 4000

$a_n=500(2)^{4-1}$

5. Find the fifth term, the tenth term, and the nth term of the arithmetic sequence.

4, 7, 10, 13, ...

$d=3$; $a_5=16$

$a_n=a_1+(n-1)d$

$a_{10}=4+(10-1)3$

$=4+27$

$=31$

$a_{10}=31$

$a_n=4+(n-1)3$

$=4+3n-3$

$a_n=1+3n$

6. Find the sum of the arithmetic sequence Sn that satisfies the stated conditions.

$a_7 = \frac{2}{3}$, $d = -\frac{2}{3}$, $n = 27$

$a_7 = a_1 + (n-1)d$

$\frac{2}{3} = a_1 + (7-1)\left(-\frac{2}{3}\right)$

$\frac{2}{3} = a_1 - 4$

$\frac{14}{3} = a_1$

$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$ or $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

$= \frac{27}{2}\left[2\left(\frac{14}{3}\right) + (27-1)\left(-\frac{2}{3}\right)\right]$

$= \frac{27}{2}\left(2\left(\frac{14}{3}\right) + 26\left(-\frac{2}{3}\right)\right] = -108$

7. Find the sum.

$\sum_{n=1}^{22}(3n + 4)$

$a_1 = 7$

$a_2 = 10$

$a_3 = 13$

$d = 3$

$a_1 = 7$

$n = 22$

$S_n = \frac{22}{2}\left[2(7) + (22-1)3\right]$

$= 11\left[14 + 63\right] = 847$

8. Find the sum.

$\sum_{n=1}^{28}\left(\frac{1}{2}n + 6\right)$

$a_1 = \frac{13}{2}$

$a_2 = 7$

$a_3 = \frac{15}{2}$

$d = \frac{1}{2}$

$n = 28$

$a_1 = \frac{13}{2}$

$S_n = \frac{28}{2}\left[2\left(\frac{13}{2}\right) + (28-1)\left(\frac{1}{2}\right)\right]$

$= 14\left[13 + \frac{27}{2}\right]$

$S_n = 371$

9. A contest will have five cash prizes totaling $9,000, and there will be a $400 difference between successive prizes. Find the first prize.

$S_5 = 9000$

$d = -400$

$9000 = \frac{5}{2}\left[2a_1 + (5-1)(-400)\right]$

$\left(\frac{2}{5}\right)9000 = \frac{5}{2}(2a_1 + -1600)$

$3600 = 2a_1 - 1600$

$5200 = 2a_1$ $\quad 2600 = a_1$

2600, 2200, 1800, 1400, 1000

1st

10. Assuming air resistance is negligible, a small object that is dropped from a hot air balloon falls 19 feet during the first second, 57 feet during the second second, 95 feet during the third second, 133 feet during the fourth second, and so on. Find an expression for the distance the object falls in n seconds.

$19, 57, 95, 133, \ldots$

$d = 38$

total distance $= \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$= \frac{n}{2}\left(2(19) + (n-1)38\right) = \frac{n}{2}\left(\cancel{38} + 38n - \cancel{38}\right)$

$\frac{n}{2}(38n) = 19n^2$

11. Find the seventh term of the geometric sequence.

$1, -\frac{x}{2}, \frac{x^2}{4}, -\frac{x^3}{8}, \ldots$

$a_7 = a_1 r^{n-1}$

$= 1\left(-\frac{x}{2}\right)^{7-1}$

$a_7 = \frac{x^6}{2^6} = \frac{x^6}{64}$

12. Express the sum in terms of summation notation.

2 - 4 + 8 - 16 + 32 - 64

$\sum_{n=1}^{6} a_1 r^{n-1} = \sum_{n=1}^{6} 2(-2)^{n-1}$

13. Insert two geometric means between 9 and 243.

9, ___, ___, 243

27 81

$a_n = a_1 r^{n-1}$

$\frac{243}{9} = \frac{9r^{4-1}}{9}$

$27 = r^3$; $r = 3$

14. Rewrite as an expression that does not contain factorials.

$\frac{(5n+2)!}{(5n)!}$

$\frac{(5n+2)(5n+1)\cancel{(5n)!}}{\cancel{(5n)!}} = (5n+2)(5n+1)$

15. Use the binomial theorem to expand and simplify.

$(2x - y)^3$

$\binom{3}{0}(2x)^3(-y)^0 + \binom{3}{1}(2x)^2(-y)^1 + \binom{3}{2}(2x)^1(-y)^2 + \binom{3}{3}(2x)^0(-y)^3$

$(1)(8x^3)(1) + (3)(4x^2)(-y) + 3(2x)(y^2) + 1(1)(-y^3)$

$8x^3 - 12x^2y + 6xy^2 - y^3$

16. Use Pascal's Triangle to simplify.

$(m + n)^4$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$m^4 + 4m^3n + 6m^2n^2 + 4mn^3 + n^4$